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A new criterion for global robust stability of interval delayed neural networks

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Abstract

A novel criterion for the global robust stability of Hopfield-type interval neural networks with delay is presented. An example showing the effectiveness of the present criterion is given.

MSC: 34C25; 34K13; 34K25; 37N35; 37N40; 93C10; 93D05; 93D09; 93D20

Keywords: Dynamical interval neural networks; Equilibrium analysis; Global robust stability; Hopfield neural networks; Neural networks; Nonlinear systems; Stability; Time-delay systems

1. Introduction

This paper deals with the delayed neural network model defined by the following state equations:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \boldsymbol{u} \tag{1}$$

or

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau)) + u_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^{\mathrm{T}}$ is the state vector associated with the neurons, $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix ($c_i > 0, i = 1, 2, \ldots, n$), $\boldsymbol{A} = (a_{ij})_{n\times n}$ and $\boldsymbol{B} = (b_{ij})_{n\times n}$ are the connection weight and the delayed connection weight matrices, respectively, $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^{\mathrm{T}}$ is a constant external input vector, τ is the transmission delay, the f_j, $j = 1, 2, \ldots, n$, are the activation functions, $\boldsymbol{f}(\boldsymbol{x}(\cdot)) = \begin{bmatrix} f_1(x_1(\cdot)) & f_2(x_2(\cdot)) & \cdots & f_n(x_n(\cdot)) \end{bmatrix}^{\mathrm{T}}$, and the superscript 'T' to any vector (or matrix) denotes the transpose of that vector (or matrix). The activation functions are assumed to satisfy the following restrictions:

$$\left| f_j(\xi) \right| \le M_j \quad \forall \xi \in R;\ M_j > 0,\ j = 1, 2, \ldots, n \tag{3}$$

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and

$$0 \leq \frac{f_j(\xi_1) - f_j(\xi_2)}{\xi_1 - \xi_2} \leq L_j \quad j = 1, 2, \ldots, n \tag{4}$$

for each $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$, where L_j are positive constants. Such activation functions ensure the existence of an equilibrium point for Eq. (1) [1]. The quantities c_i, a_{ij}, and b_{ij} may be considered as intervalized as follows:

$$\boldsymbol{C}_I := [\underline{\boldsymbol{C}}, \overline{\boldsymbol{C}}] = \left\{\boldsymbol{C} = \operatorname{diag}(c_i) : \underline{\boldsymbol{C}} \leq \boldsymbol{C} \leq \overline{\boldsymbol{C}},\ \text{i.e., } \underline{c}_i \leq c_i \leq \bar{c}_i,\ i = 1, 2, \ldots, n\right\} \tag{5}$$

$$\boldsymbol{A}_I := [\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \underline{\boldsymbol{A}} \leq \boldsymbol{A} \leq \overline{\boldsymbol{A}},\ \text{i.e., } \underline{a}_{ij} \leq a_{ij} \leq \bar{a}_{ij}, i, j = 1, 2, \ldots, n\right\} \tag{6}$$

$$\boldsymbol{B}_I := [\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}] = \left\{\boldsymbol{B} = (b_{ij})_{n\times n} : \underline{\boldsymbol{B}} \leq \boldsymbol{B} \leq \overline{\boldsymbol{B}},\ \text{i.e., } \underline{b}_{ij} \leq b_{ij} \leq \bar{b}_{ij}, i, j = 1, 2, \ldots, n\right\}. \tag{7}$$

Definition. The system given by Eq. (1) with the parameter ranges defined by Eqs. (5)–(7) is globally robustly stable if the unique equilibrium point $\boldsymbol{x}^* = \begin{bmatrix} x_1^* & x_2^* & \cdots & x_n^* \end{bmatrix}^{\mathrm{T}}$ of the system is globally asymptotically stable for all $\boldsymbol{C} \in \boldsymbol{C}_I, \boldsymbol{A} \in \boldsymbol{A}_I, \boldsymbol{B} \in \boldsymbol{B}_I$.

In the following, $\boldsymbol{F} > \boldsymbol{0}$ denotes that the matrix $\boldsymbol{F}$ is symmetric positive definite. If $\boldsymbol{W}$ is a matrix, its norm $\|\boldsymbol{W}\|_2$ is defined as $\|\boldsymbol{W}\|_2 = \sup\{\, \|\boldsymbol{W}\boldsymbol{x}\| : \|\boldsymbol{x}\| = 1\} = \sqrt{\lambda_{\max}(\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W})}$, where $\lambda_{\max}(\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W})$ denotes the maximum eigenvalue of $\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}$.

The problem of global asymptotic stability of Eq. (1) has generated considerable interest. For a sample of literature on the subject, the reader is referred to [1–28] and the references cited therein. The problem of global robust stability of Eq. (1) with the intervalized parameters given by Eqs. (5)–(7) has also received considerable attention [1–13]. In this paper, we present a novel criterion for the global robust stability of Eq. (1) with Eqs. (5)–(7). An example showing the effectiveness of the present criterion is given.

2. The criterion

Define

$$\boldsymbol{A}^* = \left(a_{ij}^*\right)_{n\times n} = (\overline{\boldsymbol{A}} + \underline{\boldsymbol{A}})/2, \qquad \boldsymbol{A}_* = \left(a_{*ij}\right)_{n\times n} = (\overline{\boldsymbol{A}} - \underline{\boldsymbol{A}})/2. \tag{8}$$

Let the interval given by Eq. (6) be divided into the following two equal intervals:

$$\boldsymbol{A}_I^{\mathrm{I}} := [\underline{\boldsymbol{A}}, \boldsymbol{A}^*] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \underline{\boldsymbol{A}} \leq \boldsymbol{A} \leq \boldsymbol{A}^*,\ \text{i.e., } \underline{a}_{ij} \leq a_{ij} \leq a_{ij}^*, i, j = 1, 2, \ldots, n\right\} \tag{9}$$

$$\boldsymbol{A}_I^{\mathrm{II}} := [\boldsymbol{A}^*, \overline{\boldsymbol{A}}] = \left\{\boldsymbol{A} = (a_{ij})_{n\times n} : \boldsymbol{A}^* \leq \boldsymbol{A} \leq \overline{\boldsymbol{A}},\ \text{i.e., } a_{ij}^* \leq a_{ij} \leq \overline{a}_{ij}, i, j = 1, 2, \ldots, n\right\}. \tag{10}$$

Define the symmetric matrix $\boldsymbol{S}^{\mathrm{I}} = \left\{s_{ij}^{\mathrm{I}}\right\}_{n\times n}$ as

$$s_{ij}^{\mathrm{I}} = \begin{Bmatrix} -2a_{ii}^*, & \text{if } i = j \\ -\hat{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \quad \hat{a}_{ij} = \max\left\{\left|a_{ij}^* + a_{ji}^*\right|, \left|\underline{a}_{ij} + \underline{a}_{ji}\right|\right\} \tag{11}$$

and the symmetric matrix $\boldsymbol{S}^{\mathrm{II}} = \left\{s_{ij}^{\mathrm{II}}\right\}_{n\times n}$ as

$$s_{ij}^{\mathrm{II}} = \begin{Bmatrix} -2\overline{a}_{ii}, & \text{if } i = j \\ -\tilde{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \quad \tilde{a}_{ij} = \max\left\{\left|\overline{a}_{ij} + \overline{a}_{ji}\right|, \left|a_{ij}^* + a_{ji}^*\right|\right\}. \tag{12}$$

The main result is given in the following theorem.

Theorem 1. *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if there are positive constants* β_1 *and* β_2 *satisfying the following:*

$$\boldsymbol{Q}^{\mathrm{I}} = \begin{bmatrix} 2\dfrac{c_m}{L_M}\boldsymbol{I} + \boldsymbol{S}^{\mathrm{I}} - \beta_1\boldsymbol{I} & -(\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2)\boldsymbol{I} \\ -(\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2)\boldsymbol{I} & \beta_1\boldsymbol{I} \end{bmatrix} > \boldsymbol{0}, \tag{13}$$

$$\boldsymbol{Q}^{\mathrm{II}} = \begin{bmatrix} 2\dfrac{c_m}{L_M}\boldsymbol{I} + \boldsymbol{S}^{\mathrm{II}} - \beta_2\boldsymbol{I} & -(\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2)\boldsymbol{I} \\ -(\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2)\boldsymbol{I} & \beta_2\boldsymbol{I} \end{bmatrix} > \boldsymbol{0}, \tag{14}$$

where $\boldsymbol{B}^* = (\overline{\boldsymbol{B}} + \underline{\boldsymbol{B}})/2$, $\boldsymbol{B}_* = (\overline{\boldsymbol{B}} - \underline{\boldsymbol{B}})/2$, $c_m = \min_i\{\underline{c}_i\}$, $L_M = \max_i\{L_i\}$, *and* $\boldsymbol{I}$ *denotes the* $n \times n$ *identity matrix.*

Proof. Consider the problem of global robust stability of Eq. (1) with Eqs. (5), (9) and (7) and also the problem of global robust stability of Eq. (1) with Eqs. (5), (10) and (7). Clearly, solving these two problems is equivalent to solving the original problem of global robust stability of Eq. (1) with Eqs. (5)–(7). Note that if $\boldsymbol{A}$ belongs to the set $\boldsymbol{A}_I$ given by Eq. (6), then this means that $\boldsymbol{A}$ belongs to either the set $\boldsymbol{A}_I^{\mathrm{I}}$ given by Eq. (9) or the set $\boldsymbol{A}_I^{\mathrm{II}}$ given by Eq. (10). Following [9], if there is a $\beta_1 > 0$ satisfying Eq. (13), then Eq. (1) with Eqs. (5), (9) and (7) is globally robustly stable and if there is a $\beta_2 > 0$ satisfying Eq. (14), then Eq. (1) with Eqs. (5), (10) and (7) is globally robustly stable. This completes the proof of Theorem 1. □

Remark. Eq. (13) is linear in the unknown constant β_1 (free parameter), i.e., Eq. (13) is a linear matrix inequality (LMI). Similarly, Eq. (14) is a LMI. These LMIs can be solved using the celebrated LMI toolbox [29,30] which has the built-in feature of guaranteeing the existence or nonexistence, whichever is the case, of the prevailing unknown constants.

3. Example

Consider a second-order DNN characterized by

$$\overline{\boldsymbol{A}} = \begin{bmatrix} -1 & -2 \\ 0 & -2 \end{bmatrix}, \qquad \underline{\boldsymbol{A}} = \begin{bmatrix} -2 & -3 \\ -2.2 & -3 \end{bmatrix}, \qquad \overline{\boldsymbol{B}} = \underline{\boldsymbol{B}} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\overline{\boldsymbol{C}} = \underline{\boldsymbol{C}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad L_1 = L_2 = 1. \tag{15}$$

We will first apply some existing criteria [5–9]. The main results of [5–9] are the following:

Theorem 2 ([5]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if*

$$\boldsymbol{S} > \boldsymbol{0}, \tag{16a}$$

$$2\frac{c_m}{L_M} - 1 - (\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2)^2 \geq 0, \tag{16b}$$

where the symmetric matrix $\boldsymbol{S} = \{s_{ij}\}_{n\times n}$ *is defined by*

$$s_{ij} = \begin{Bmatrix} -2\overline{a}_{ii}, & \text{if } i = j \\ -\breve{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \qquad \breve{a}_{ij} = \max\left\{ \left|\overline{a}_{ij} + \overline{a}_{ji}\right|, \left|\underline{a}_{ij} + \underline{a}_{ji}\right| \right\}. \tag{17}$$

Theorem 3 ([6]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if there are a positive diagonal matrix* $\boldsymbol{P} = \mathrm{diag}(p_1, p_2, \ldots, p_n)$, $p_1 > 0$, $p_2 > 0, \ldots, p_n > 0$, *and a positive definite matrix* $\boldsymbol{D} = \boldsymbol{D}^{\mathrm{T}} = \{d_{ij}\}_{n\times n} > \boldsymbol{0}$ *such that*

$$\boldsymbol{T} > \boldsymbol{0}, \tag{18a}$$

$$(\|\boldsymbol{B}_*\|_2 + \|\boldsymbol{B}^*\|_2)^2 \leq (2\beta - \|\boldsymbol{D}\|_2) / \left(\|\boldsymbol{P}\|_2^2 \left\|\boldsymbol{D}^{-1}\right\|_2\right), \tag{18b}$$

where $\beta = \min_i \left\{ p_i \underline{c}_i / L_i \right\}$ *and* $\boldsymbol{T} = \left\{ t_{ij} \right\}_{n \times n}$ *is a symmetric matrix defined by*

$$t_{ij} = \begin{Bmatrix} -2p_i \overline{a}_{ii}, & \text{if } i = j \\ -\widehat{a}_{ij}, & \text{if } i \neq j \end{Bmatrix}, \quad \widehat{a}_{ij} = \max \left\{ \left| p_i \overline{a}_{ij} + p_j \overline{a}_{ji} \right|, \left| p_i \underline{a}_{ij} + p_j \underline{a}_{ji} \right| \right\}. \tag{19}$$

Theorem 4 ([7]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if there are a positive diagonal matrix* $\boldsymbol{P} = \mathrm{diag}(p_1, p_2, \ldots, p_n)$*,* $p_1 > 0,\ p_2 > 0, \ldots, p_n > 0$*, and a positive definite matrix* $\boldsymbol{D} = \boldsymbol{D}^{\mathrm{T}} = \left\{ d_{ij} \right\}_{n \times n} > \boldsymbol{0}$ *such that*

$$\begin{bmatrix} 2\dfrac{c_m}{L_M}\boldsymbol{P} - \boldsymbol{PU} - \boldsymbol{U}^{\mathrm{T}}\boldsymbol{P} - \boldsymbol{D} & -\boldsymbol{PV} \\ -\boldsymbol{V}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{D} \end{bmatrix} > \boldsymbol{0}, \tag{20}$$

where $\boldsymbol{U} = \left\{ u_{ij} \right\}_{n \times n}$ *is a matrix defined by*

$$u_{ij} = \begin{Bmatrix} \overline{a}_{ii}, & \text{if } i = j \\ \max\left(\left| \underline{a}_{ij} \right|, \left| \overline{a}_{ij} \right| \right), & \text{if } i \neq j \end{Bmatrix} \tag{21}$$

and the matrix $\boldsymbol{V} = \left\{ v_{ij} \right\}_{n \times n}$ *is given by*

$$v_{ij} = \max\left(\left| \underline{b}_{ij} \right|, \left| \bar{b}_{ij} \right| \right). \tag{22}$$

Theorem 5 ([8]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if there is a positive diagonal matrix* $\boldsymbol{P} = \mathrm{diag}(p_1, p_2, \ldots, p_n)$*,* $p_1 > 0,\ p_2 > 0, \ldots, p_n > 0$*, such that*

$$2\beta \boldsymbol{I} + \boldsymbol{T} - 2 \left\| \boldsymbol{P} \right\|_2 (\left\| \boldsymbol{B}^* \right\|_2 + \left\| \boldsymbol{B}_* \right\|_2) \boldsymbol{I} > \boldsymbol{0}. \tag{23}$$

Theorem 6 ([8]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if*

$$(c_m / L_M) - \left(\left\| \boldsymbol{A}^* \right\|_2 + \left\| \boldsymbol{A}_* \right\|_2 + \left\| \boldsymbol{B}^* \right\|_2 + \left\| \boldsymbol{B}_* \right\|_2 \right) > 0. \tag{24}$$

Theorem 7 ([9]). *Under the conditions given by Eqs.* (3)–(7)*, Eq.* (1) *is globally robustly stable if there is a positive constant* β *such that*

$$\begin{bmatrix} 2\dfrac{c_m}{L_M}\boldsymbol{I} + \boldsymbol{S} - \beta \boldsymbol{I} & -(\left\| \boldsymbol{B}^* \right\|_2 + \left\| \boldsymbol{B}_* \right\|_2)\boldsymbol{I} \\ -(\left\| \boldsymbol{B}^* \right\|_2 + \left\| \boldsymbol{B}_* \right\|_2)\boldsymbol{I} & \beta \boldsymbol{I} \end{bmatrix} > \boldsymbol{0}. \tag{25}$$

For the example given by Eq. (15), the matrix $\boldsymbol{S}$ becomes

$$\boldsymbol{S} = \begin{bmatrix} 2 & -5.2 \\ -5.2 & 4 \end{bmatrix}. \tag{26}$$

Clearly, Eq. (16a) is violated in this example. The matrix $\boldsymbol{T}$ becomes

$$\boldsymbol{T} = \begin{bmatrix} 2p_1 & -|3p_1 + 2.2p_2| \\ -|3p_1 + 2.2p_2| & 4p_2 \end{bmatrix}. \tag{27}$$

From Eq. (27) one can see that Eq. (18a) is violated for all $p_1 > 0,\ p_2 > 0$ in this example. For this example, one has

$$2\frac{c_m}{L_M}\boldsymbol{P} - \boldsymbol{PU} - \boldsymbol{U}^{\mathrm{T}}\boldsymbol{P} = \begin{bmatrix} 4p_1 & -3p_1 - 2.2p_2 \\ -3p_1 - 2.2p_2 & 6p_2 \end{bmatrix}. \tag{28}$$

Note that $2(c_m/L_M)\boldsymbol{P} - \boldsymbol{PU} - \boldsymbol{U}^{\mathrm{T}}\boldsymbol{P} > \boldsymbol{0}$ is a necessary condition for Eq. (20) to hold. From Eq. (28) it is easy to observe that there do not exist $p_1 > 0$, $p_2 > 0$ to satisfy the condition $2(c_m/L_M)\boldsymbol{P} - \boldsymbol{PU} - \boldsymbol{U}^{\mathrm{T}}\boldsymbol{P} > \boldsymbol{0}$ in this example. In this example, one obtains

$$2\beta\boldsymbol{I} + \boldsymbol{T} = \begin{bmatrix} 4p_1 & -|3p_1 + 2.2p_2| \\ -|3p_1 + 2.2p_2| & 2p_1 + 4p_2 \end{bmatrix}, \quad \text{if } p_1 \leq p_2 \tag{29}$$

and

$$2\beta\boldsymbol{I} + \boldsymbol{T} = \begin{bmatrix} 2p_1 + 2p_2 & -|3p_1 + 2.2p_2| \\ -|3p_1 + 2.2p_2| & 6p_2 \end{bmatrix}, \quad \text{if } p_2 \leq p_1. \tag{30}$$

Note that $2\beta\boldsymbol{I} + \boldsymbol{T}$ is required to be positive definite for Eq. (23) to hold. From Eqs. (29) and (30) it can easily be verified that there do not exist $p_1 > 0$, $p_2 > 0$ to satisfy the condition $2\beta\boldsymbol{I} + \boldsymbol{T} > \boldsymbol{0}$ in this example. In the example under consideration, $(c_m/L_M) = 1$, $\|\boldsymbol{A}^*\|_2 = 3.987$, $\|\boldsymbol{A}_*\|_2 = 1.383$, $\|\boldsymbol{B}^*\|_2 = 0.1$, $\|\boldsymbol{B}_*\|_2 = 0$. Clearly, Eq. (24) is violated in this example. Finally, one obtains

$$2\frac{c_m}{L_M}\boldsymbol{I} + \boldsymbol{S} = \begin{bmatrix} 4 & -5.2 \\ -5.2 & 6 \end{bmatrix}. \tag{31}$$

For Eq. (25) to hold $2\,(c_m/L_M)\,\boldsymbol{I} + \boldsymbol{S}$ should necessarily be positive definite, which clearly is not the case in this example.

Summarizing the above, each of Theorems 2–7 fails to verify the global robust stability in this example.

Let $\beta_1 = \beta_2 = 0.5$. Then the matrices $\boldsymbol{Q}^{\mathrm{I}}$ and $\boldsymbol{Q}^{\mathrm{II}}$ in Eqs. (13) and (14) become

$$\boldsymbol{Q}^{\mathrm{I}} = \begin{bmatrix} 4.5 & -5.2 & -0.1 & 0 \\ -5.2 & 6.5 & 0 & -0.1 \\ -0.1 & 0 & 0.5 & 0 \\ 0 & -0.1 & 0 & 0.5 \end{bmatrix} \tag{32}$$

and

$$\boldsymbol{Q}^{\mathrm{II}} = \begin{bmatrix} 3.5 & -3.6 & -0.1 & 0 \\ -3.6 & 5.5 & 0 & -0.1 \\ -0.1 & 0 & 0.5 & 0 \\ 0 & -0.1 & 0 & 0.5 \end{bmatrix}, \tag{33}$$

respectively. The matrices in Eqs. (32) and (33) are positive definite. Thus Theorem 1 affirms the global robust stability in this example.

For numerical simulation, we consider two specific examples presented below.

Example 1. Consider the following model belonging to Eq. (15):

$$\boldsymbol{A} = \begin{bmatrix} -1.5 & -2.5 \\ -1.1 & -2.5 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad L_1 = L_2 = 1, \tag{34}$$

where $f_i(x) = (1/2)(|x+1| - |x-1|)$ $(i = 1, 2)$. The following three cases are given: case 1 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.2, 0.3)$ for $t \in [-1.0, 0]$; case 2 with the initial state $(\varphi_1(t), \varphi_2(t)) = (-1.0, 1.0)$ for $t \in [-1.0, 0]$; case 3 with the initial state $(\varphi_1(t), \varphi_2(t)) = (-0.1, 0.6)$ for $t \in [-1.0, 0]$; Fig. 1 depicts the time responses of the state variables of $x_1(t)$ and $x_2(t)$ with step $h = 0.05$ and input vector $\boldsymbol{u} = (1.0, 0.5)^{\mathrm{T}}$. It confirms that the proposed criterion leads to the unique stable solution for the model.

Example 2. Consider the model given by

$$\boldsymbol{A} = \begin{bmatrix} -1 & -3 \\ -2.2 & -2 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad L_1 = L_2 = 1, \tag{35}$$

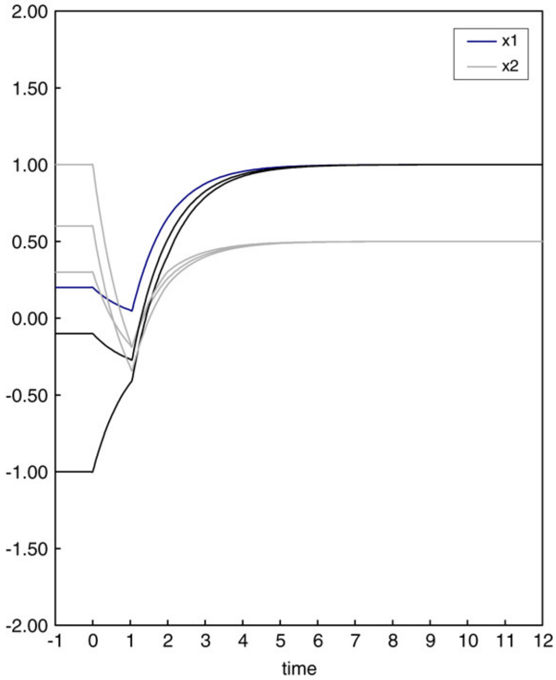

Fig. 1. Transient response of state variable $x_1(t)$ and $x_2(t)$ in Example 1.

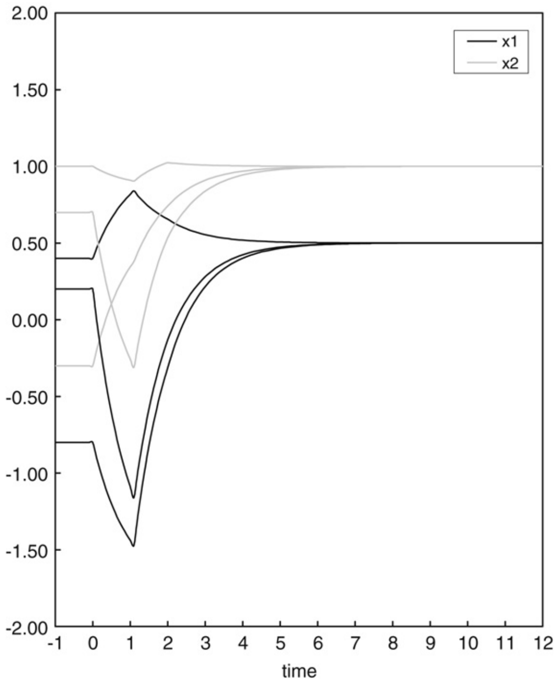

Fig. 2. Transient response of state variables $x_1(t)$ and $x_2(t)$ in Example 2.

which belongs to Eq. (15). Let $f_i(x) = (1/2)(|x+1| - |x-1|)$ $(i = 1, 2)$. The following three cases are considered: case 1 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.4, -0.3)$ for $t \in [-1.0, 0]$; case 2 with the initial state $(\varphi_1(t), \varphi_2(t)) = (-0.8, 1.0)$ for $t \in [-1.0, 0]$; case 3 with the initial state $(\varphi_1(t), \varphi_2(t)) = (0.2, 0.7)$ for $t \in$

$[-1.0,\ 0]$; Fig. 2 depicts the time responses of the state variables of $x_1(t)$ and $x_2(t)$ with step $h = 0.1$ and input vector $\boldsymbol{u} = (0.5,\ 1.0)^{\mathrm{T}}$. It confirms that the proposed criterion leads to the unique stable solution for the model.

4. Conclusion

A criterion for the global robust stability of a class of interval DNNs has been presented. The criterion is in marked contrast to the existing criteria. The usefulness of the criterion for obtaining new global robust stability results is illustrated with the help of an example.

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